Lecture 8 – 05/11/2025

The *p-n* junction

- Basic considerations
- At thermal equilibrium
- Space charge region

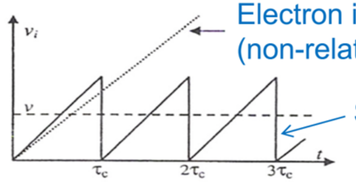
Summary Lecture 7: Carrier Transport

Low electric field

Thermal equilibrium → scattering,
characteristic time τ_c ~ 1 ps

Mobility: $\mu = \frac{q\tau_c}{m^*}$

Electron in vacuum (non-relativistic case)

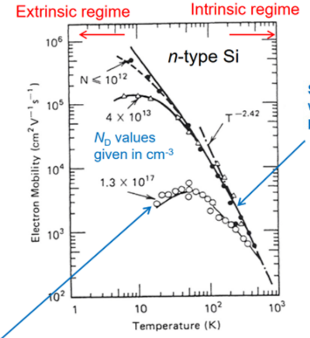


Sawtooth behavior

$<v> = v_d = \mu E$

Mobility vs doping

Extrinsic regime — Intrinsic regime



Very doped semiconductors have an optimal temperature for conductivity

Scattering due to interactions with the lattice (mainly LA and LO phonons, interband scattering, ...)

Ionized impurities (Coulomb interaction) = $f(v_{th}, N_{D,A})$

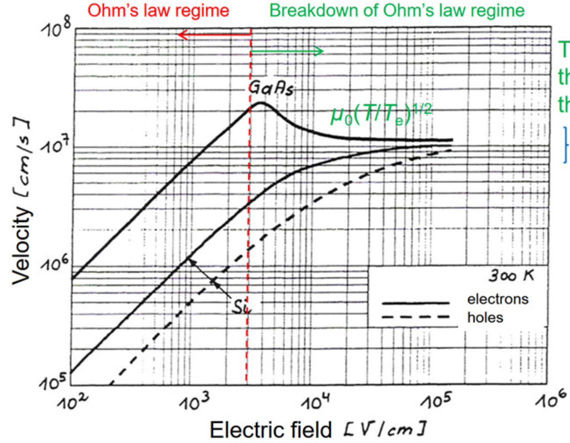
Inverses of mobility add up (which means that resistivities add up)

$$1/\mu_{tot} = 1/\mu_{latt} + 1/\mu_{ions} + \ldots$$ Matthiessen rule

High electric field

Effective temperature: $1/2\ m^* v_e^2 = 3/2\ k_B T_e$ $\qquad \mu = \mu_0 \sqrt{\frac{T}{T_e}}$

Ohm's law regime — Breakdown of Ohm's law regime



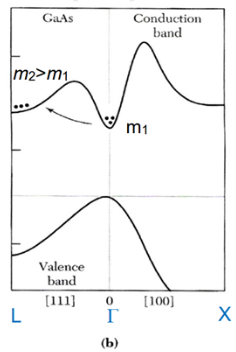
The larger the electric field E, the larger T_e until reaching the saturation velocity

Saturation velocity

Energy stored by electrons is released through an optical phonon of energy E_{ph}

$$v_{sat} = (E_{ph}/m^*)^{1/2}$$

In some materials, high E changes the population distribution → resistance changes



Summary Lecture 7: Out of Equilibrium Semiconductors

Continuity equations

Impact of excess carriers for electrons and holes

$$\frac{\partial n}{\partial t} = G - R + \left(\frac{1}{q}\right)\nabla \cdot \mathbf{J_n}$$

Electron generation / recombination

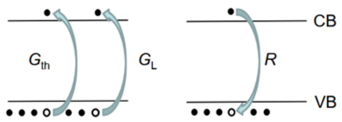
Mass action law

$$R = Bnp$$

B: Bimolecular recombination coefficient

Lifetime of electrons in the case of small deviations to thermal equilibrium

$$\tau_n = 1/(Bp_0)$$



Weak injection: excess carrier concentration much smaller than the equilibrium majority carrier concentration.

$$\frac{\partial n}{\partial t} = \left(\frac{1}{q}\right)\nabla \cdot \mathbf{J_n} - \frac{(n - n_0)}{\tau_n} + G_L, \quad \text{and} \quad \frac{\partial p}{\partial t} = \frac{\partial n}{\partial t}$$

Band-to-band recombinations

Weak carrier concentration

$$R - G_{th} = B(n_0 + \Delta n)\cdot(p_0 + \Delta p) - Bn_0p_0 \approx Bp_0 \cdot \Delta n \approx \frac{\Delta n}{\tau_n}.$$

$$\tau_n = \frac{1}{Bp_0} \text{ for electrons in a } p\text{-type semiconductor}$$

and similarly,

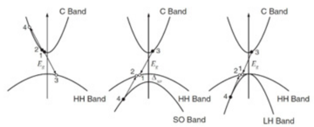
$$\tau_p = \frac{1}{Bn_0} \text{ for holes in an } n\text{-type semiconductor}$$

High carrier concentration: Auger-Meitner process

$$\tau_{n,\text{Auger}} \approx \frac{1}{\left[C\cdot(p_0 + \Delta n)^2\right]}, \text{ high injection } n \approx \Delta n$$

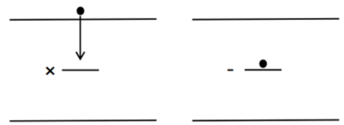
$$\tau_{p,\text{Auger}} \approx \frac{1}{\left[C\cdot(n_0 + \Delta p)^2\right]}, \text{ high injection } p \approx \Delta p$$

Auger coefficient



Single level recombinations

Consider an intermediate level that can trap electrons and two charges (either hole or electron)



4 types of transition

Capture of an electron of the conduction band

$$r_{c,n} = \beta_n n N_t^x = v_{th}\sigma_n n N_t^x$$

Emission of an electron toward the conduction band

$$r_{e,n} = e_n N_t^-$$

Capture of a hole from the valence band

$$r_{c,p} = \beta_p p N_t^- = v_{th}\sigma_p p N_t^-$$

Emission of a hole toward the valence band

$$r_{e,p} = e_p N_t^x$$

Emission probabilities at thermal equilibrium

$$e_n = v_{th}\sigma_n n_i \exp\left[\left(E_t - E_F\right)/k_BT\right] = v_{th}\sigma_n n_t$$

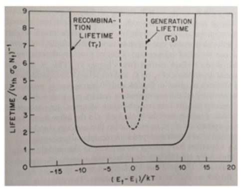
$$e_p = v_{th}\sigma_p n_i \exp\left[\left(E_F - E_t\right)/k_BT\right] = v_{th}\sigma_p p_t$$

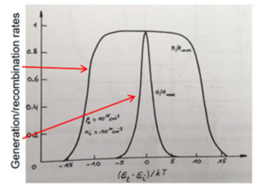
Under weak injection

$$R = \sigma_n\sigma_p v_{th} N_t \frac{np - n_t p_t}{\sigma_n(n + n_t) + \sigma_p(p + p_t)}$$

Case of a *p*-type semiconductor, weak injection

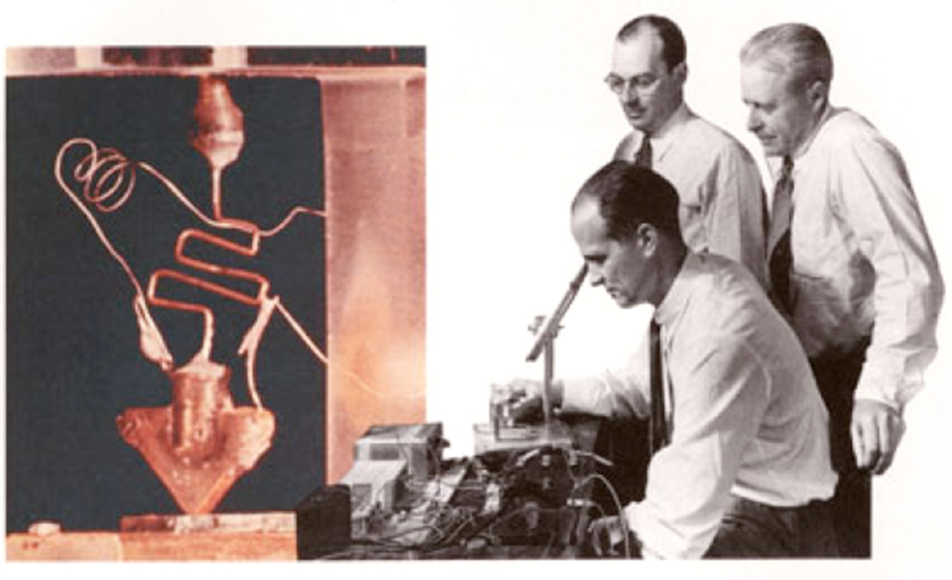
$$R = \sigma_n v_{th} N_t \Delta n = \frac{\Delta n}{\tau_n} \text{ with } \tau_n = \frac{1}{\sigma_n v_{th} N_t}$$





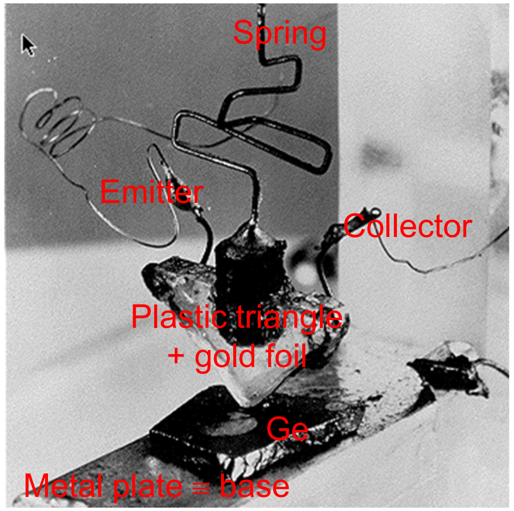
1st *p-n* junction and 1st transistor (1947)

First transistor   Bardeen, Shockley and Brattain 1947



1939

Nobel prize in physics 1956



Point contact transistor

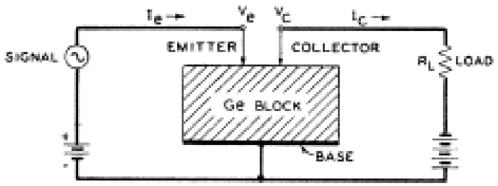
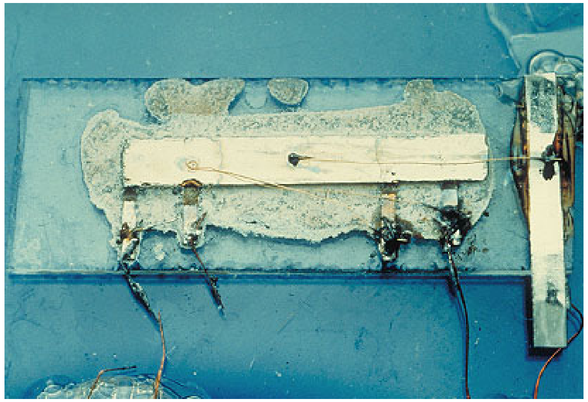


FIG. 1. Schematic of semi-conductor triode.

J. Bardeen and W. H. Brattain, Phys. Rev. **74**, 230 (1948)

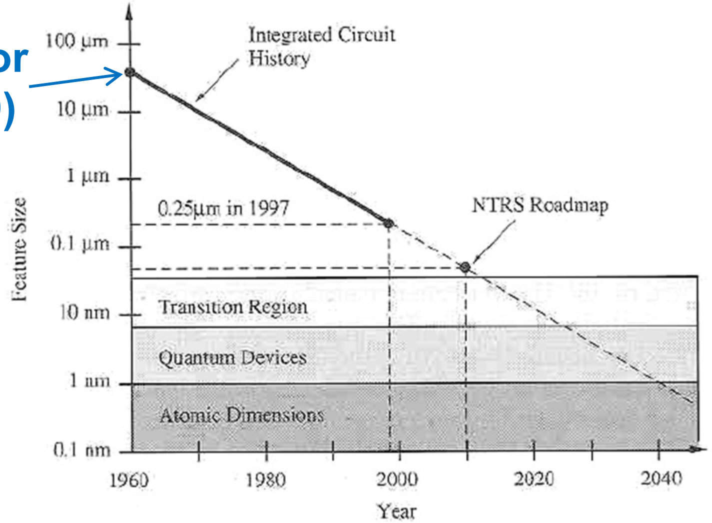
> 550 citations

Transistor: past... and ...future

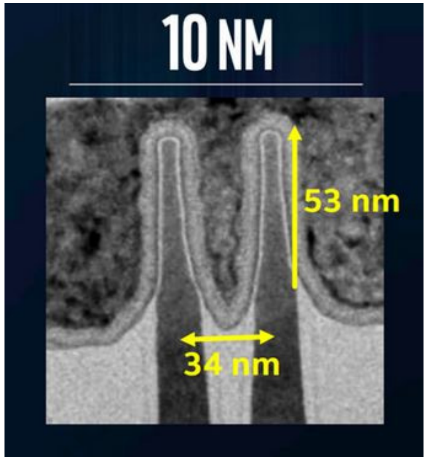


First integrated circuit (few transistors)
Jack Kilby, TI - 1958 – Nobel prize (2000)
Dimension: 11 × 1.6 mm²

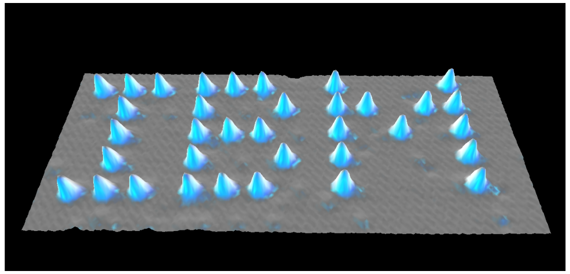
CMOS transistor invention (1960)



National technology roadmap for semiconductors (NTRS)
Now ITRS (I: international)

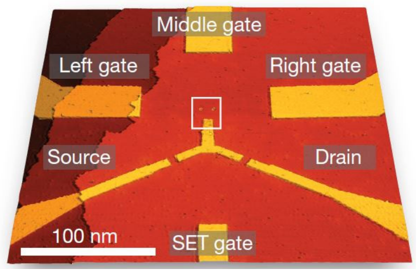


Intel 10 nm process node



Manipulation of individual atoms

Realization of qubit gates

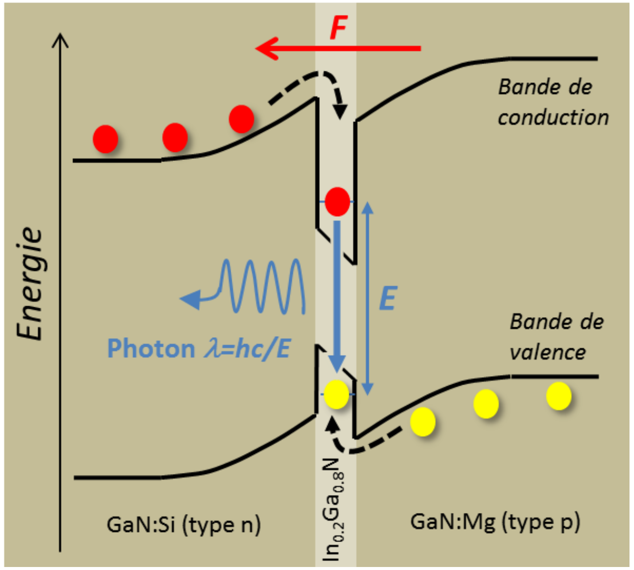


Y. He *et al*., Nature **571**, 371 (2019)
> 260 citations

p-n junction

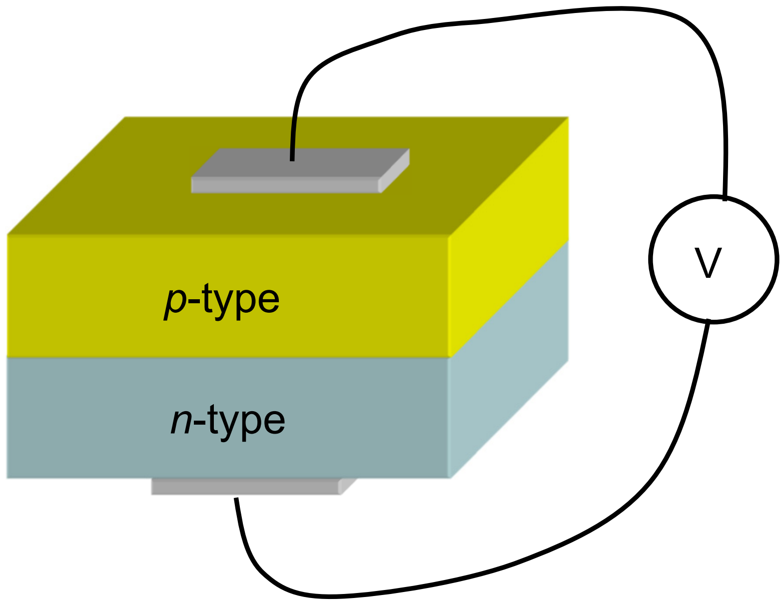
Some applications

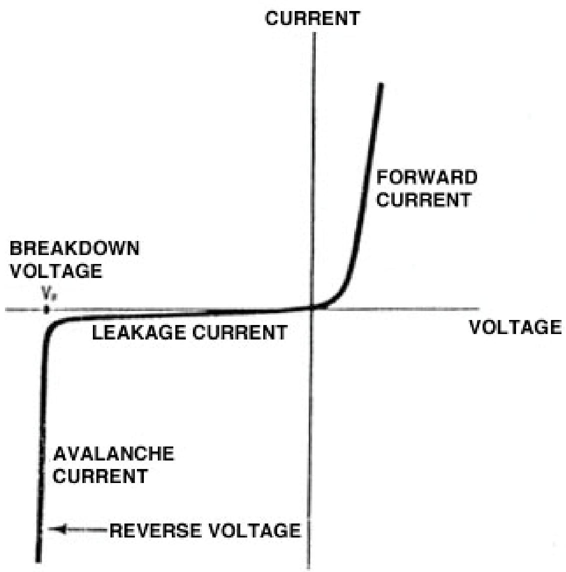
- Bipolar transistors
- LEDs
- Laser diodes
- Solar cells
- Photodetectors



p-n junction (2D-geometry)

Made of two adjacent semiconductor layers which are *p*-type and *n*-type doped





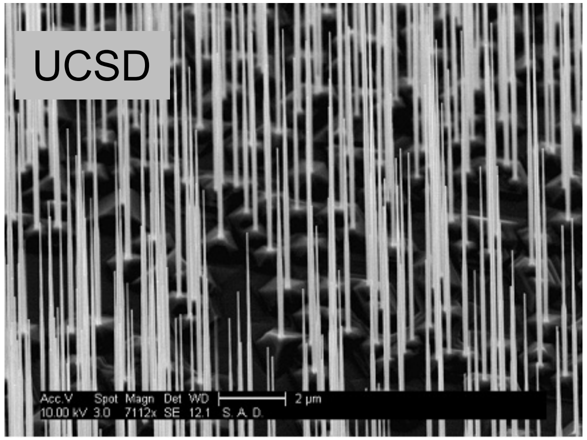
A p-n junction behaves like a diode (rectifying behavior)

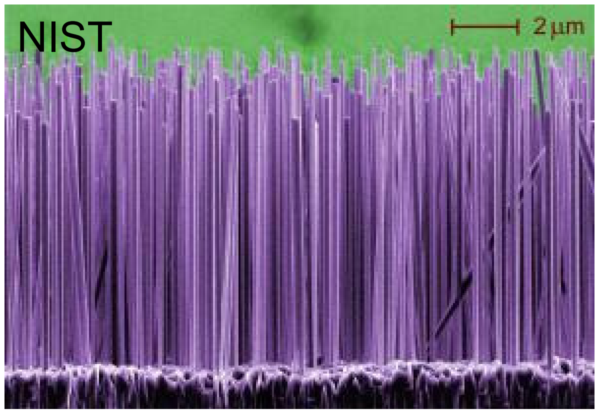
Early historical account detailed in: W. Shockley, Bell Syst. Tech. J. **28**, 435 (1949); C.-T. Sah, R. N. Noyce, and W. Shockley, Proc. IRE **45**, 1228 (1957) > 1740 citations

> 2250 citations

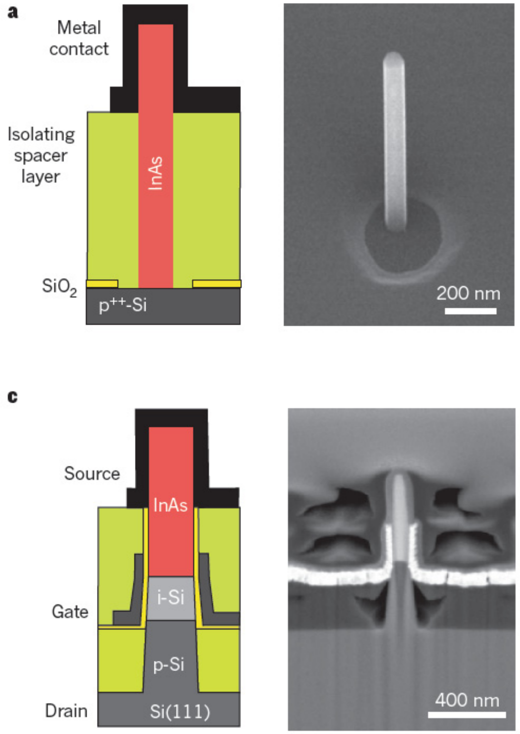
p-n junction (research, nanowire geometry)

Semiconductor nanowires (candidates for tunnel-FETs)





Subthreshold swing < 60 mV/decade at 300 K
(cf. forthcoming Lecture 9)

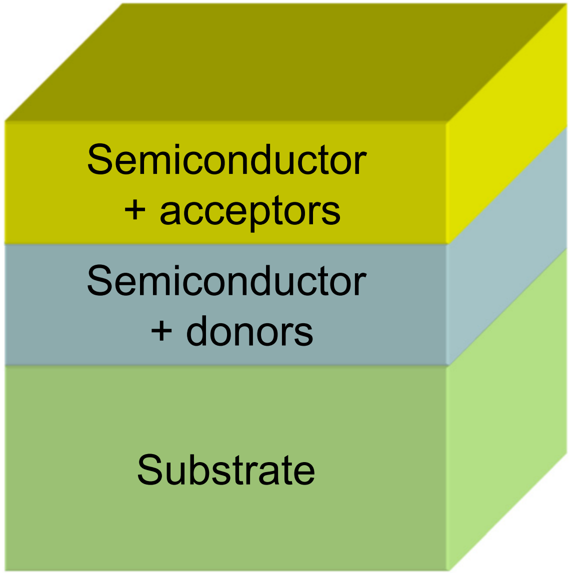


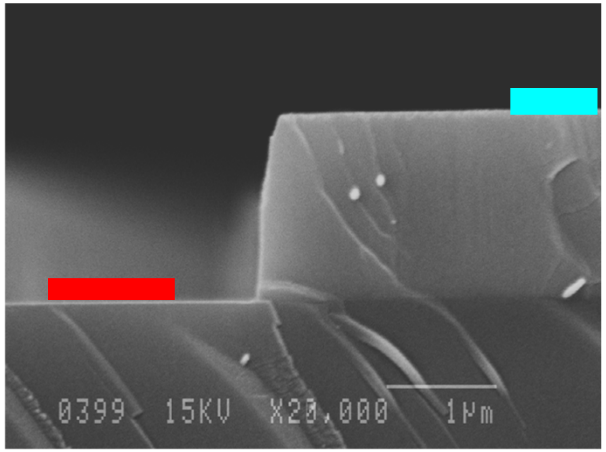
A. M. Ionescu and H. Riel, Nature **479**, 329 (2011)

> 2420 citations

p-n junction fabrication

During growth by impurity incorporation

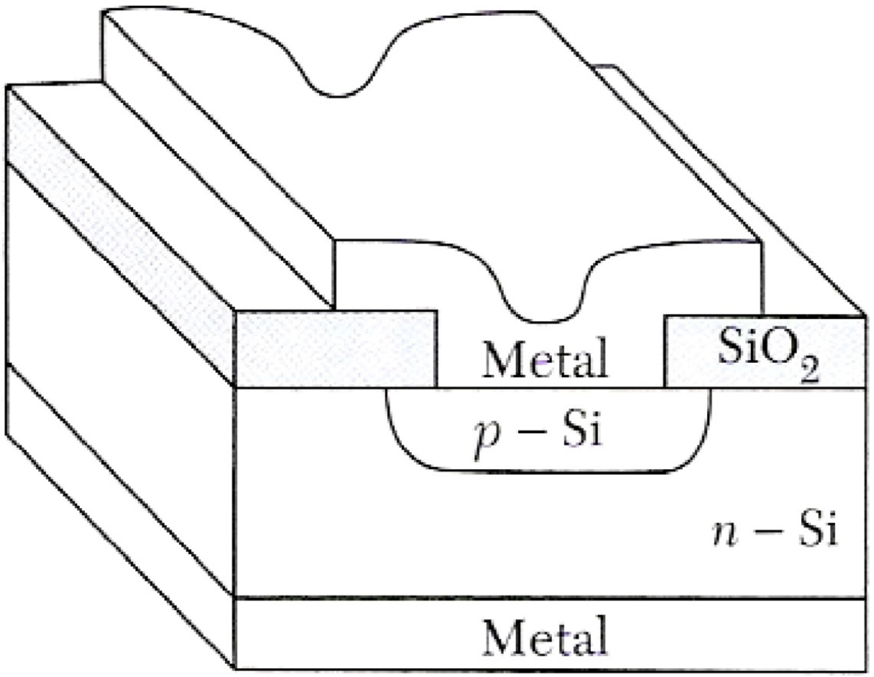




SEM image of a *p-n* junction (LED)

p-n junction fabrication

Post-growth: by impurity diffusion or implantation



p-n junction fabrication

Post-growth: by impurity diffusion or implantation

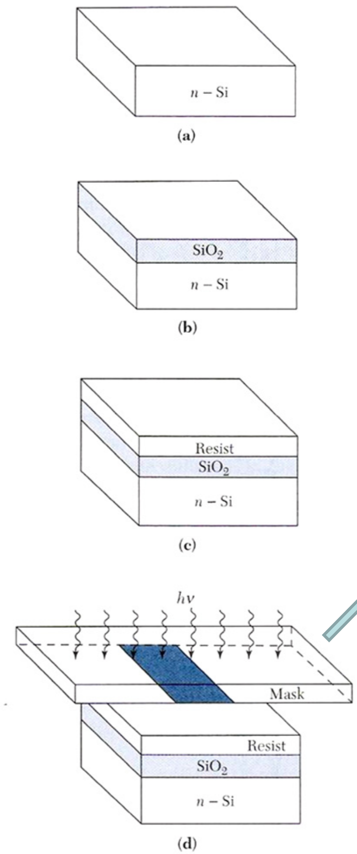


Fig. 1 (*a*) A bare *n*-type Si wafer. (*b*) An oxidized Si wafer by dry or wet oxidation. (*c*) Application of resist. (*d*) Resist exposure through the mask.

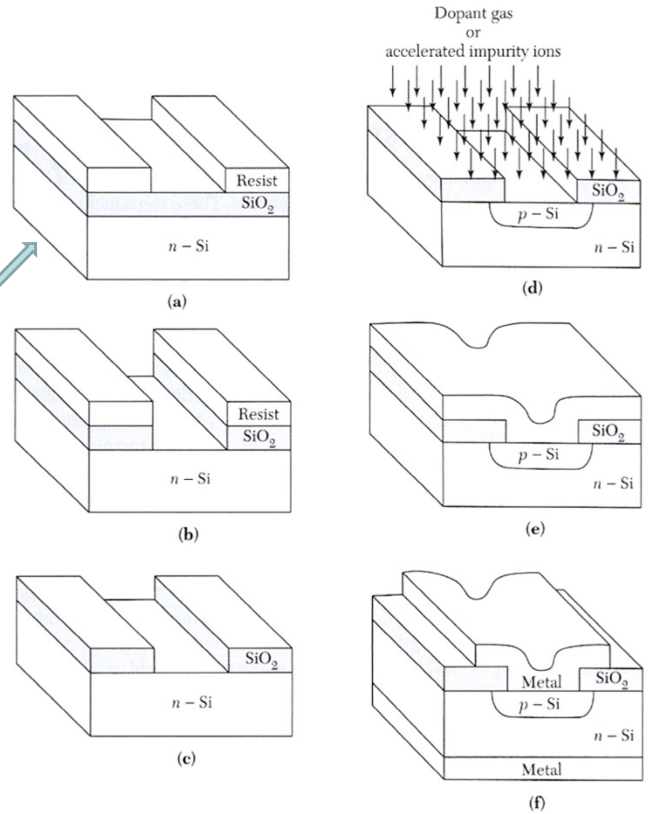
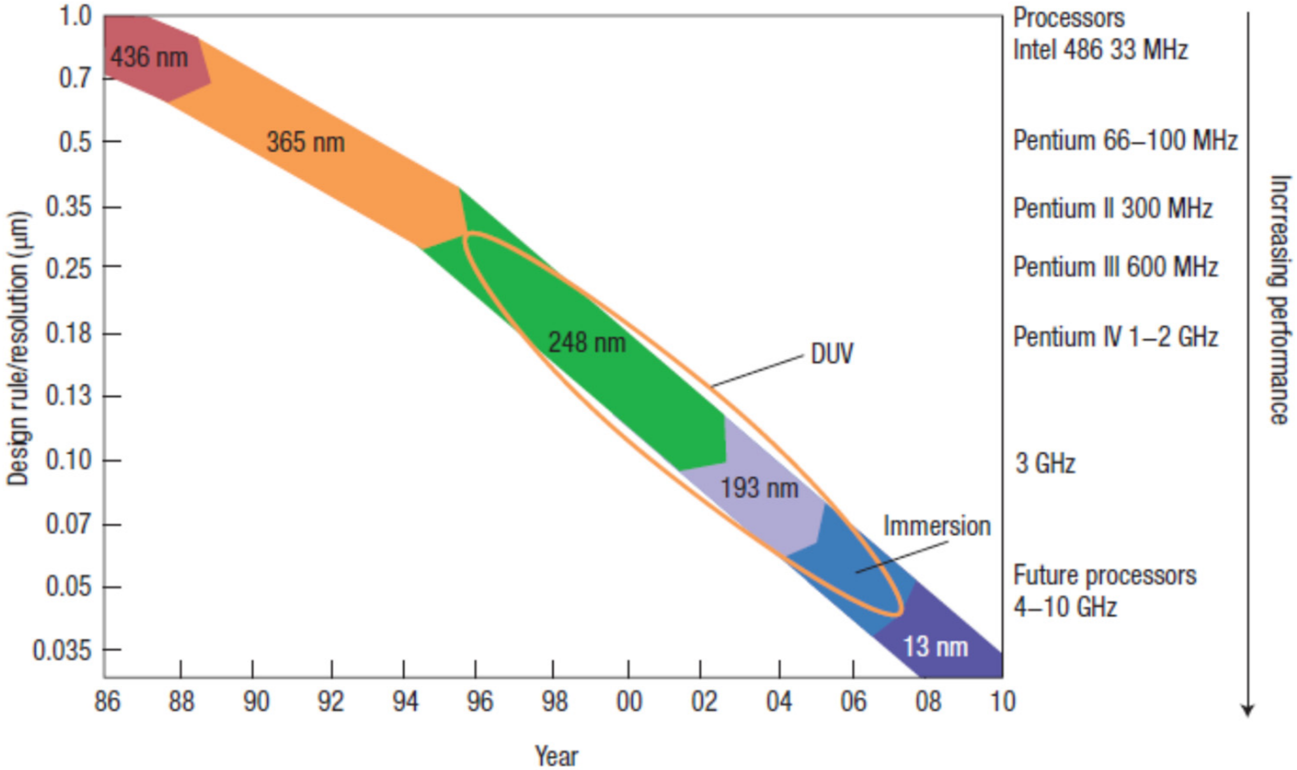


Fig. 2 (*a*) The wafer after the development. (*b*) The wafer after SiO_2 removal. (*c*) The final result after a complete lithography process. (*d*) A *p–n* junction is formed in the diffusion or implantation process. (*e*) The wafer after metallization. (*f*) A *p–n* junction after the complete processes.

Optical lithography



Speed/power tradeoff ⇒ CPU underclocking can save a lot of power while sacrificing much less the performance (motivation for multicore CPUs), $P \propto CV^2 f$

Example: Intel chips (2008), single core underclocking by 20% will save half the power for 13% less performance !

Optical lithography

The **fabrication of integrated circuits** (ICs) relies on **expensive photolithography systems**

Patterns optically imaged onto Si wafers covered with a photoresist

- Leading company: ASML (Dutch), >3/4 of the market $\Rightarrow$ provider of immersion and EUV lithography machines (sole supplier of EUV tools, market capitalization in 2024 ~US$400 billion)
- Other players: Canon, Nikon, Ultratech (now part of Veeco Instr.) (USA)

Optical lithography machines set the transistor technology node, i.e., the typical half-pitch ($\equiv$ half distance between identical features in an array) for a memory cell.
22 nm technology node for the CMOS process, e.g., multicore processors. 2 nm technology node introduced by TSMC in 2025 (Gate-All-Around nanosheet transistors, 3 nm family already in high-volume production)

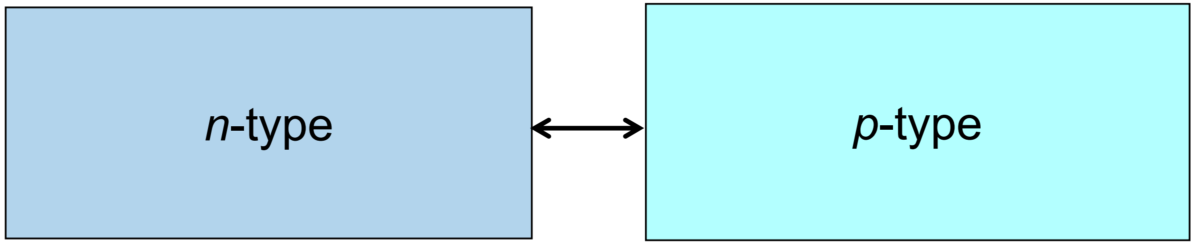
Optical lithography

New generation of wafer stepper:

- Lens-free optical lithography
- All-mirror-based technology operating in the extreme UV (13.5 nm), hence under vacuum (to avoid air absorption)
- Used by Intel, IBM, Samsung and TSMC
- Cost up to 200 M$/unit for the Twinscan NXE:3600D

EUV lithography scanner – Twinscan NXE:3400

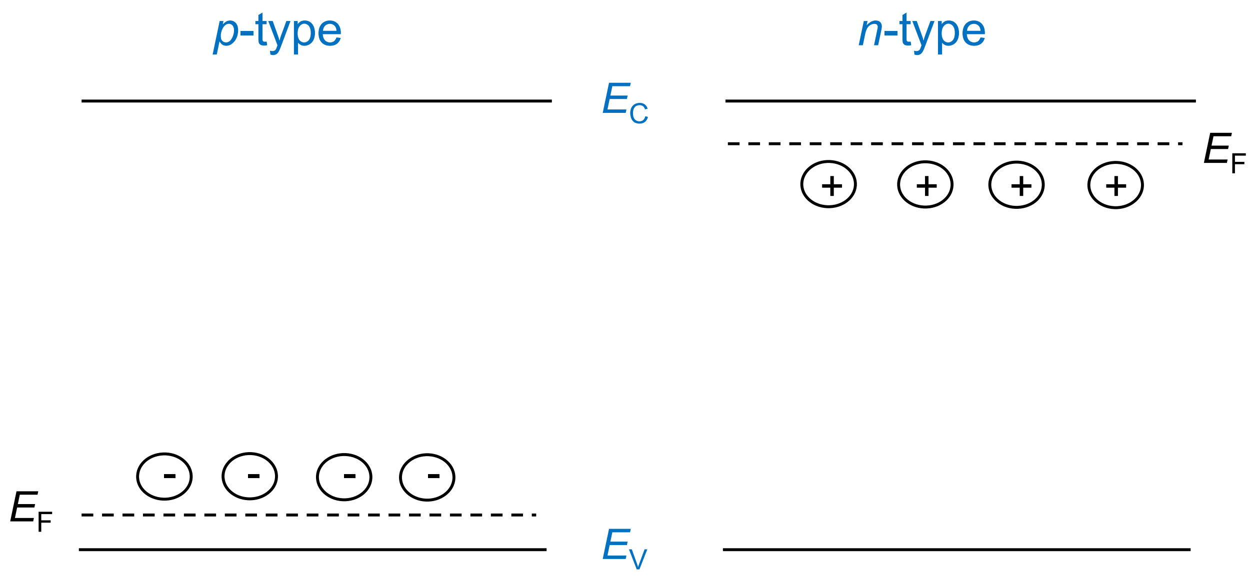
p-*n* junction: how does it work?



Case of the abrupt *p*-*n* junction (metallurgical junction)

Band diagram

- **At thermal equilibrium**



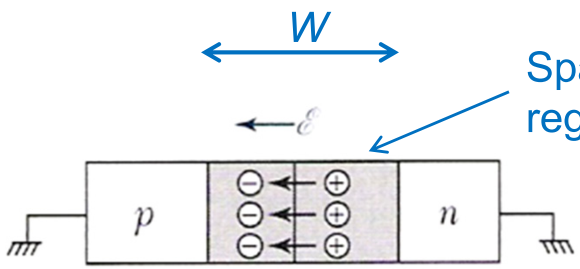
- Concentration gradients ⇒ diffusion currents
- Uncompensated ionized impurities ⇒ built-in electric field ⇒ drift currents

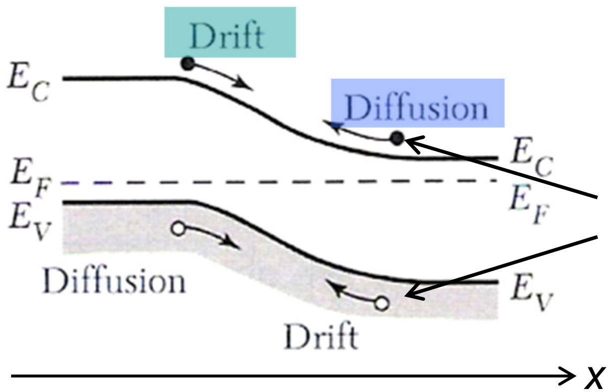
p-n junction at equilibrium

- Diffusion and drift currents

$\mathbf{J}_{n,drift} = \sigma_n \mathbf{E} = e\mu_n n \mathbf{E}$

$\mathbf{J}_{n,diff} = eD_n \,\mathbf{grad}\, n$





Einstein relation:

$$\frac{D}{\mu} = \frac{k_B T}{e}$$

Diffusion coefficient in $cm^2\ s^{-1}$

Link between *number* (*j*) and *electrical* (*J*) current densities:

$$\mathbf{J_n} = -e\mathbf{j_n}$$
and
$$\mathbf{J_p} = e\mathbf{j_p}$$

Band diagram

- **At thermal equilibrium**

The Fermi level is constant throughout the structure (details to be seen in the series)

$J_n = J_{n,\mathrm{drift}} + J_{n,\mathrm{diff}} = 0$ (also true for holes)

NO "net" current flowing through the junction

To be verified in the exercises

$$J_n = e\mu_n nE + eD_n \frac{dn}{dx} = \mu_n n \frac{dE_F}{dx} = 0$$

Keep in mind that Ohm's law is valid provided $v_d << v_{th}$!

1D case

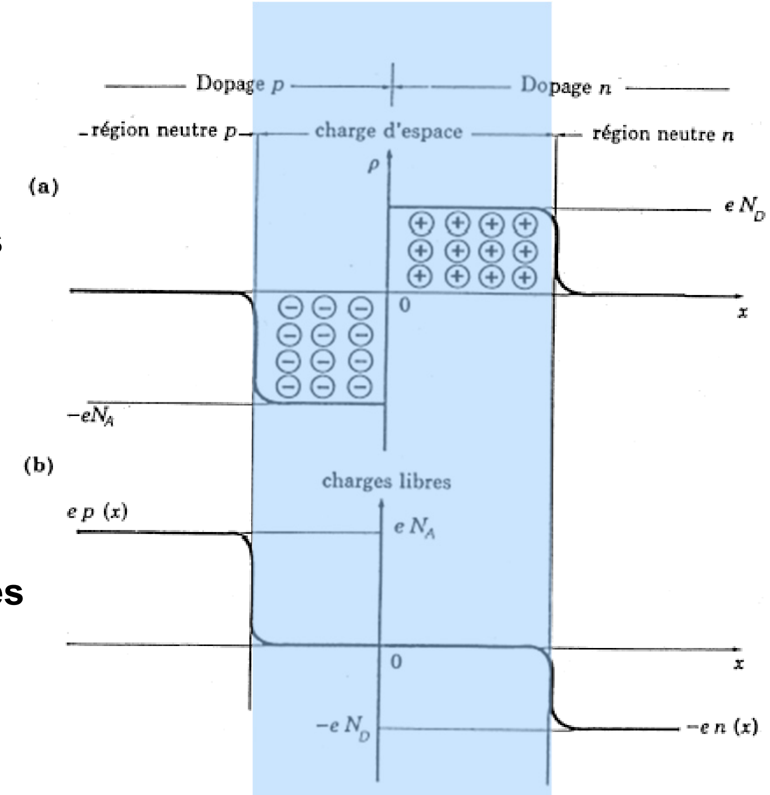
Full compensation between the drift and diffusion currents !

Band diagram

Space charge

Fixed charges

Mobile charges

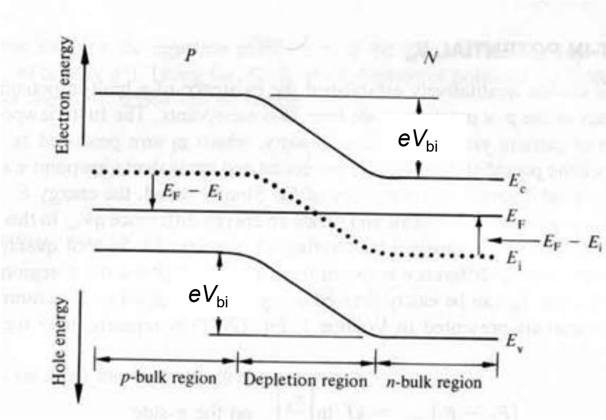


Band diagram

- **At thermal equilibrium**

Built-in potential:

$eV_{bi} = (E_F - E_i)_n - (E_F - E_i)_p$ ≡ energy loss across the junction



($n \approx N_D$ and $p \approx N_A$)

with

$$E_F = E_c - k_B T \ln\frac{N_c}{n} = E_v + k_B T \ln\frac{N_v}{p}$$

$$p = N_v \exp\left[-(E_F - E_i)/k_B T\right]\exp\left[-(E_i - E_v)/k_B T\right] = n_i \exp\left[(E_i - E_F)/k_B T\right]$$

Similar treatment for electrons

If we consider that all impurities are ionized

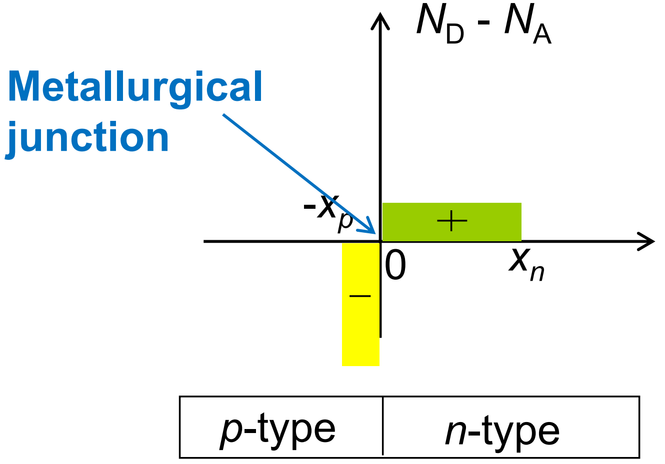
$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_v N_c}{N_A N_D}\right)$$

Built-in barrier

This built-in potential V_{bi} is the consequence of the space charge due to carrier diffusion

Space charge region

Abrupt junction Poisson's equation to be solved as a function of position:



$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_r\varepsilon_0}$$

Electrostatic potential vs position

$$\frac{d^2\phi}{dx^2} = e\frac{N_A}{\varepsilon} \quad for \quad -x_p \le x < 0$$

$$\frac{d^2\phi}{dx^2} = -e\frac{N_D}{\varepsilon} \quad for \quad 0 < x \le x_n$$

$$\varepsilon = \varepsilon_r\varepsilon_0$$

$$\int_{-x_p}^{x_n} \rho dx = 0$$

Charge neutrality: $x_p N_A = x_n N_D$

Space charge region extent: $W = x_p + x_n$

Electric field in the space charge region

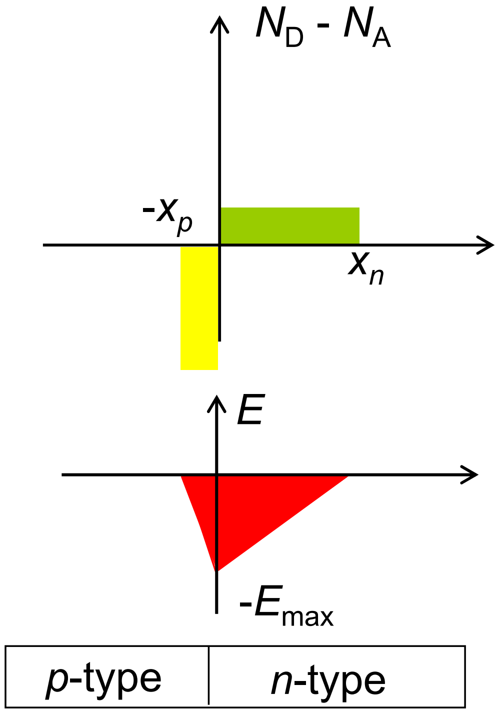
$$E(x) = -\frac{d\phi}{dx} = -e\frac{N_A(x+x_p)}{\varepsilon} \quad [-x_p, 0]$$

$$E(x) = -\frac{d\phi}{dx} = e\frac{N_D(x-x_n)}{\varepsilon} \quad [0, x_n]$$

$$|E_{max}| = e\frac{N_A x_p}{\varepsilon} = e\frac{N_D x_n}{\varepsilon}$$

Space charge region

Abrupt junction



$$V_{bi} = -\int_{-x_p}^{x_n} E(x)\,dx$$

$$= e\frac{N_A}{\varepsilon}\int_{-x_p}^{0}(x+x_p)\,dx - e\frac{N_D}{\varepsilon}\int_{0}^{x_n}(x-x_n)\,dx$$

$$= e\frac{N_A x_p^2}{2\varepsilon} + e\frac{N_D x_n^2}{2\varepsilon}$$

$$= \frac{E_{max}x_p}{2} + \frac{E_{max}x_n}{2} = \frac{1}{2}E_{max}W$$

Note that:

$$V_{bi} = \frac{1}{\varepsilon}\int_{-x_p}^{x_n}\left[\int_{-x_p}^{x}\rho\,dx\right]dx$$

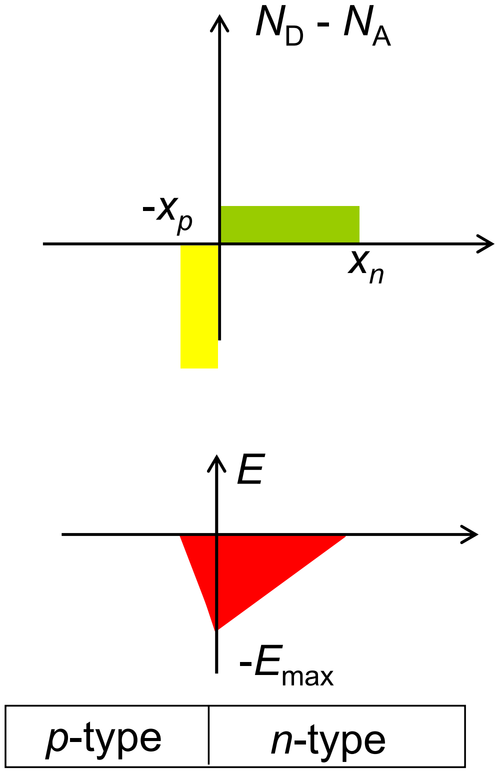
which can also be written:

Valid both at thermal equilibrium and out of equilibrium ⇒

$$V_{bi} = \frac{1}{\varepsilon}\int_{-x_p}^{x_n} x\rho\,dx$$

Space charge region

Abrupt junction



$$V_{bi} = e\frac{N_A x_p^2}{2\varepsilon} + e\frac{N_D x_n^2}{2\varepsilon}$$

Charge neutrality: $x_p N_A = x_n N_D$

Space charge extent: $W = x_p + x_n$

⇨ $W = x_p\,(1+N_A/N_D) = x_p(N_D+N_A)/N_D$

$W = x_n \ldots.$

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}$$

All the parameters entering into this expression are known beforehand!

Extent of the space charge region

Space charge region

Example: silicon-based *p-n* junction at 300 K

n-type: $N_D = 10^{18}$ cm^{-3} and $p = n_i^2/N_D = 10^2$ cm^{-3}

p-type: $N_A = 10^{18}$ cm^{-3} and $n = n_i^2/N_A = 10^2$ cm^{-3}

Cf. Lecture 6 ⇒ use of mass action law at thermal equilibrium + full ionization of impurities (+ charge neutrality condition)

$N_c = 2.7 \times 10^{19}$ cm^{-3} and $N_v = 1.1 \times 10^{19}$ cm^{-3}

We find:

$eV_{bi} = 0.96$ eV

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_v N_c}{N_A N_D}\right)$$

$W = 50$ nm

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}

$\varepsilon_r = 11.9$

$e = 1.6 \times 10^{-19}$ C

$E_{max} = 3.8 \times 10^5$ V cm^{-1}

$$E_{max} = 2V_{bi}/W$$

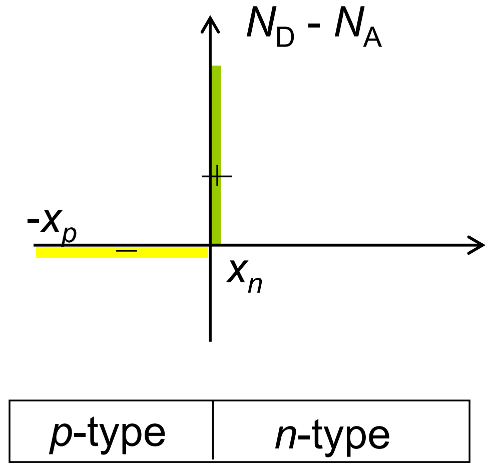
Other example: $N_A = N_D = 10^{15}$ cm^{-3}, $eV_{bi} = 0.61$ eV, $W = 1.2$ µm

Space charge region

Space charge in the *n*-type and *p*-type regions

$x_n = N_A/(N_D+N_A)W$

$x_p = N_D/(N_D+N_A)W$



If $N_D >> N_A$ then

$$W = \sqrt{\frac{2\varepsilon}{e}\frac{V_{bi}}{N_A}}$$

and $x_p = W$

One-sided abrupt junction approximation

The space charge mostly extends in the less doped region